Math Strategies Geometry

Little Chute
Math
Grades 9-11, .5 Credit

Course Overview/Description

Geometry explores the properties of points, lines, planes, and solids. This course also includes the different methods of proof and logic and investigates parallel lines, planes, congruent and similar polygons, trigonometry, circles, and coordinate geometry.

Scope and Sequence

Timeframe	Unit	Instructional Topics
7.5 days	1. Shapes & Transformations	1.1 Building Blocks 1.2 Transformations 1.3 Shapes
5.5 days	2. Angles & Measurement	2.1 Angle Relationships 2.2 Area Formulas 2.3 Distance
6.5 days	3. Justification & Similarity	3.1 Similarity 3.2 Similarity Proofs
6.5 days	4. Trigonometry & Probability	4.1 Slope Ratio 4.2 Probability Models
7.5 days	5. More Trigonometric Ratios	5.1 Trig Ratios 5.2 Special Right Triangles 5.3 Law of Sines/Cosines
6.5 days	6. Congruent Triangles	6.1 Congruent Triangles 6.2 Problem Solving
9 days	7. Proof & Quadrilaterals	7.1 Problem Solving 7.2 Properties of Quadrilaterals 7.3 Coordinate Geometry
7 days	8. Polygons & Circles	8.1 Polygons

		8.2 Area Ratios 8.3 Area & Circumference of Circles
5.5 days	9. Solids & Constructions	9.1 3-D Solids 9.2 Constructions
9 days	10. Circles & Conditional Probability	10.1 Relationships in a Circle 10.2 Conditional Probability 10.3 Permutations & Combinations
7 days	11. Solids & Circles	11.1 Volume & Surface Area 11.2 Spheres
5 days	12. Conics & Closure	12.1 Conic Sections 12.2 Problem Solving

UNIT 1: Shapes and Transformations

Duration of Unit: 7.5 Day(s)

Description of Unit: Students become familiar with basic geometric shapes and learn how to describe each one using its attributes, such as parallel sides or rotation symmetry. They investigate three basic rigid transformations: reflection (flip), rotation (turn), and translation (slide).

Essential Questions and/or Enduring Understandings:
1.1 Building Blocks
1.2 Transformations
1.3 Shapes

ESSENTIAL Standards	Learning Targets
1.1	Students will work together to build symmetrical designs using the same basic shapes.
	Students will generate questions to investigate, make predictions, and test their predictions as they work with Möbius strips and related constructions

	Students will investigate how the perimeter and area of a shape change as the shape is enlarged proportionally.
	Students will be introduced to how to develop a convincing argument.
	Students will build an understanding of what an angle is and how it is measured.
1.2	Students will use their spatial visualization skills to investigate reflection.
	Students will understand the three rigid transformations (translations, reflections, and rotations) and will learn some connections between them.
	Students will recognize that the slopes of parallel lines are the same and the slopes of perpendicular lines are opposite reciprocals.
	Students will learn how to translate a geometric figure on a coordinate grid and that reflection and reflection symmetry can help them discover relationships within a shape, namely an isosceles triangle.
	Students will use what they know about transformations to make other shapes including: rhombus, square, parallelogram, isosceles triangle, right triangle, kite, and dart.
1.3	Students will learn how to classify shapes by their attributes using Venn diagrams.
	Students will continue to study the attributes of shapes as they begin to formalize their vocabulary: both names of shapes (such as quadrilateral and trapezoid) and attributes of shapes (such as parallel sides and right angle).
NICE TO KNOW Standards	**Learning Targets**

UNIT 2: Angles and Measurement
Duration of Unit: 5.5 Day(s)
Description of Unit: Students will further investigate how to describe a complex figure by developing ways to accurately determine its angles,

area, and perimeter. They will also use transformations from Chapter 1 to uncover special relationships between angles within a figure. Throughout this chapter students will be asked to solve problems, such as those involving area or angles, in more than one way. This will require them to "see" shapes in multiple ways and to gain a broader understanding of problem solving.

Essential Questions and/or Enduring Understandings:
2.1 Angle Relationships
2.2 Area Formulas
2.3 Distance

ESSENTIAL Standards	Learning Targets
2.1	Students will learn how to name angles, and will learn the three main relationships for angle measures, namely, supplementary, complementary, and same (have the same measure).
	Students will use their understanding of translation to determine that when a transversal intersects parallel lines, corresponding angles have equal measure.
	Students will apply their knowledge of corresponding angles, and will develop theorems about alternate interior and same-side interior angles.
	Students will discover that the angles in a triangle add up to 180°.
	Students will learn the converses of some of their angle theorems, and see arguments for them.
2.2	Students will gain a geometric sense of length and area by investigating various unit measures of each concept.
	Students will learn how to find the area of a triangle and will develop multiple methods to find the area of composite shapes formed by rectangles and triangles.
	Students will use rectangles and triangles to develop algorithms to find the area of new shapes, including parallelograms and trapezoids.
	Students will explore how to find the height of a triangle given that one side has been specified as the base.
	Students will find the areas of composite shapes using what they have learned about the areas of triangles, parallelograms, and trapezoids.
2.3	Students will also learn how to determine whether or not three given lengths can make a triangle.

	Students will understand how to find the maximum and minimum lengths of a third side given the lengths of the two other sides.
	Students will develop and prove the Pythagorean Theorem.
NICE TO KNOW Standards	**Learning Targets**

UNIT 3: Justification and Similarity	
Duration of Unit: 6.5 Day(s)	
Description of Unit: Students will focus on comparing; they will explore ways to determine if two figures have the same shape (that is, they are similar). They will also develop ways to use the information about one figure to learn more about another that has the same shape. Making logical and convincing arguments that support specific ideas about the shapes students are studying is another important skill. In this chapter, they will learn how they can document facts to support a conclusion in a flowchart.	
Essential Questions and/or Enduring Understandings: 3.1 Similarity 3.2 Similarity Proofs	
ESSENTIAL Standards	**Learning Targets**
3.1	Students will learn about the concept of dilation and will investigate the characteristics that figures share if they have the same shape.
	Students will learn that figures that can be related through a sequence of transformations that include a dilation are similar and will determine that multiplying (and dividing) lengths of figures by a common number (zoom factor) produces a similar figure.
	Students will examine the ratio of the perimeters of similar figures and will practice setting up and solving equations to solve proportional problems.

	Students will apply proportional reasoning and will learn how to write similarity statements.
3.2	Students will learn the SAS ~ and AA ~ conditions for determining triangle similarity.
	Students will learn how to use flowcharts to organize their arguments for triangle similarity and will continue to practice applying the AA ~ and SAS ~ conditions.
	Students will investigate the fact that if two triangles are similar and the common ratio between the lengths of their corresponding sides is 1, then the triangles must be congruent.
	Students will complete their list of triangle similarity conditions involving sides and angles, learning about the SSS ~ condition in the process.
	Students will practice using the three triangle similarity conditions (AA ~, SAS ~, and SSS ~) and organizing their reasoning in a flowchart.
NICE TO KNOW Standards	**Learning Targets**

UNIT 4: Trigonometry & Probability	
Duration of Unit: 6.5 Day(s)	
Description of Unit: Students will discover that the side ratios in a right triangle can serve as a powerful mathematical tool that allows them to find missing side lengths and the missing angle measures for any right triangle. They will also learn how these ratios (called trigonometric ratios) can be used in solving problems. Students will develop additional prediction skills as they extend their understanding of probability. They will examine different models to represent possibilities and to assist them in calculating probabilities.	
Essential Questions and/or Enduring Understandings: 4.1 Slope Ratio 4.2 Probability Models	
ESSENTIAL	**Learning Targets**

Standards	
4.1	Students will recognize that all the slope triangles on a given line are similar to each other and will begin to connect a specific slope to a specific angle measurement and ratio.
	Students will use technology to generate slope ratios for new angles in order to solve for missing side lengths of triangles.
4.2	Students will learn how to use a probability area model to represent a situation of chance
	Students will further the difference between theoretical and experimental probability.
	Students will use tree diagrams and area models as ways to represent and solve probability problems.
	Students will learn mathematical language for calculating probabilities of unions, intersections, and complements of events.
	Students will learn how to find the expected value of a game of chance.
NICE TO KNOW Standards	**Learning Targets**

UNIT 5: More Trigonometric Ratios
Duration of Unit: 7.5 Day(s)
Description of Unit: Students will learn about other side ratio relationships using the hypotenuse that will allow them to find missing side lengths and the missing angle measures for any right triangle. In addition, they will develop tools to complete their triangle toolkit so that they can find the missing angle measures and side lengths for any triangle, provided that enough information is given. Students will then explore ways to choose an appropriate tool to solve new problems in unfamiliar contexts.
Essential Questions and/or Enduring Understandings: 5.1 Trig Ratios

5.2 Special Right Triangles
5.3 Law of Sines/Cosines

ESSENTIAL Standards	Learning Targets
5.1	Students will learn about the sine and cosine ratios and will start a Triangle Toolkit.
	Students will develop strategies to recognize which trigonometric ratio to use based on the relative position of the reference angle and the given sides involved.
	Students will understand how to use trigonometric ratios to find the unknown angle measures of a right triangle and will be introduced to the concept of “inverse.”
	Students will use sine, cosine, and tangent ratios to solve application problems.
5.2	Students will recognize the similarity ratios in 30°- 60°- 90° and 45°- 45°- 90° triangles and begin to apply those ratios as a shortcut to finding missing side lengths.
	Students will learn to recognize 3:4:5 and 5:12:13 triangles and find other examples of Pythagorean Triples.
5.3	Students will develop a method to solve for missing sides and angles for a non-right triangle.
	Students will also develop the Law of Sines and use it to find missing side lengths and angles of non-right triangles.
	Students will develop the Law of Cosines.
	Students will learn that multiple triangles are sometimes possible when two side lengths and an angle not between them are given (SSA).
NICE TO KNOW Standards	**Learning Targets**

UNIT 6: Congruent Triangles

Duration of Unit: 6.5 Day(s)

Description of Unit: Students will find ways to determine whether two triangles are congruent. They will participate in several projects and activities that will help them synthesize their understanding and make connections between different concepts they have learned so far. Students will consolidate what they know, apply it in new ways, and identify what they still need to learn.

Essential Questions and/or Enduring Understandings:
6.1 Congruent Triangles
6.2 Problem Solving

ESSENTIAL Standards	Learning Targets
6.1	Students will practice identifying congruent triangles by first determining that the triangles are similar and that the ratio of corresponding sides is 1.
	Students will use their understanding of similarity and congruence to develop conditions that guarantee that triangles are congruent (SSS ≅, ASA ≅, AAS ≅, HL ≅, and SAS ≅).
	Students will extend their use of flowcharts to document triangle congruence facts.
	Students will recognize the converse relationship between conditional statements, and will then investigate the relationship between the truth of a statement and the truth of its converse.
6.2	Students will review angle relationships, trigonometry, and similar triangles.
	Students will review area and perimeter of a triangle, trigonometry, Pythagorean Theorem, and the Triangle Angle Sum Theorem.
	Students will review building models, similarity, and inverse trigonometric ratios.

	Students will collect experimental data and construct a probability model to represent the game.
	Students will review transformations and symmetry.
NICE TO KNOW Standards	**Learning Targets**

UNIT 7: Proof and Quadrilaterals	
Duration of Unit: 9 Day(s)	
Description of Unit: Students begin with a set of explorations designed to introduce them to new geometric topics that they will explore further in Chapters 8 through 12. They will learn about the special properties of a circle, explore three-dimensional shapes, and use a hinged mirror to learn more about a rhombus. Using congruent triangles, students will explore the relationships between the sides and diagonals of a parallelogram, kite, trapezoid, rectangle, and the rhombus. As they explore new geometric properties, they will formalize their understanding of proof.	
Essential Questions and/or Enduring Understandings: 7.1 Problem Solving 7.2 Properties of Quadrilaterals 7.3 Coordinate Geometry	
ESSENTIAL Standards	**Learning Targets**
7.1	Students will explore the special properties of a circle (specifically, its constant radius and constant diameter) and will explore Reuleaux (pronounced “roo-LOW”) curves and square wheels.
	Students will review shapes and their properties as they fold a circle to create a tetrahedron.
	Students will analyze and solve several “shortest distance” problems and will reinforce their understanding of reflection and similarity.

	Students will review how to create regular polygons with a hinged mirror and will use their understanding of reflection and congruence to learn more about the central angles of these shapes.
7.2	Students will be introduced to proof and will learn more properties of parallelograms and kites.
	Students will use their understanding of congruent triangles to prove properties of rhombi and will practice using a flowchart structure to organize a proof.
	Students will continue developing flowchart proofs as a way to communicate a logical argument and will prove that all rectangles are also parallelograms.
	Students will write flowchart proofs to demonstrate additional properties of quadrilaterals and isosceles triangles.
	Students will continue to develop their skills of writing proofs as they prove new properties of triangles and quadrilaterals.
7.3	Students will investigate quadrilaterals for special properties, such as parallel sides or a right angle.
	Students will develop methods for finding the midpoint of a segment on a coordinate grid as they continue their study of coordinate geometry.
	Students will analyze quadrilaterals on a coordinate grid and identify them by type.
NICE TO KNOW Standards	**Learning Targets**

UNIT 8: Polygons & Circles
Duration of Unit: 7 Day(s)
Description of Unit: Students will broaden their understanding of polygons to include those with 5, 8, 10, and even 100 sides. They will develop a way to find the area and perimeter of a regular polygon and will study how the area and perimeter changes as the number of sides increases. Students will re-examine similar shapes to study what happens to the area and perimeter of a shape when the shape is enlarged or

reduced. Lastly, they will connect their understanding of polygons with their knowledge of the area ratios of similar figures to find the area and circumference of circles of all sizes.

Essential Questions and/or Enduring Understandings:
8.1 Polygons
8.2 Area Ratios
8.3 Area & Circumference of Circles

ESSENTIAL Standards	Learning Targets
8.1	Students will learn that regular polygons can be built using congruent isosceles triangles with certain angle measures.
	Students will learn how to find the sum of the interior angles of a polygon and will be able to apply this skill to solve problems.
	Students will learn how to determine the measure of an interior and exterior angle of a regular polygon.
	Students will develop multiple strategies to find the measures of interior and exterior angles of a regular polygon as well as the sum of the interior angles of polygons in general.
	Students will develop an algorithm to find the area of any regular polygon.
8.2	Students will learn that the ratio of the areas of similar figures is the square of the ratio of similarity (also called zoom factor).
	Students will continue developing flowchart proofs as a way to communicate a logical argument and will prove that all rectangles are also parallelograms.
	Students will continue to develop their understanding of how the area and perimeter of a shape change as the shape is enlarged or reduced proportionally.
8.3	Students will discover the area and circumference formulas for a circle with radius 1.
	Students will use their understanding of the ratios of areas of similar figures to develop a method of finding the area and circumference of a circle with any sized radius.
	Students will develop methods to find the area of sectors and the length of arcs.

	Students will use problem-solving strategies to find areas of circular and polygonal regions in context.
NICE TO KNOW Standards	**Learning Targets**

UNIT 9: Solids & Constructions	
Duration of Unit: 5.5 Day(s)	
Description of Unit: Students will turn their focus to three-dimensional shapes (called solids), such as cubes and cylinders. They will learn several ways to represent three-dimensional solids and develop methods to measure their volumes and surface areas. They will also learn how to use special tools to construct accurate diagrams of two-dimensional shapes and geometric relationships. During this investigation, they will revisit many of the geometric conjectures and theorems that they have developed so far.	
Essential Questions and/or Enduring Understandings: 9.1 3-D Solids 9.2 Constructions	
ESSENTIAL Standards	**Learning Targets**
9.1	Students will learn how to represent three-dimensional solids using side views and a mat plan.
	Students will be introduced to volume as a form of measurement.
	Students will understand how to represent a solid with a net and will be introduced to prisms.
	Students will learn how to find the surface area of a solid.
	Students will practice finding the surface area and volume of non-rectangular prisms and cylinders.
	Students will understand that the ratio of the volumes of similar figures is the cube of the linear scale factor and they will use this relationship in applications.

	Students will apply their understanding of the ratios of similarity.
9.2	Students will also learn how to construct the incenter of a triangle, a circle inscribed within a triangle, a regular hexagon, and an equilateral triangle.
	Students will understand how to construct a perpendicular bisector and an angle bisector and will understand how the properties of the diagonals of a rhombus help create each construction.
	Students will learn how to construct a line parallel to a given line through a given point not on the line and how to construct a square.
	Students will learn about the medians and centroid of a triangle and will understand how to construct them.
NICE TO KNOW Standards	**Learning Targets**

UNIT 10: Circles & Conditional Probability	
Duration of Unit: 9 Day(s)	
Description of Unit: Students will explore the relationships between angles, arcs, and chords in a circle. The focus of their work turns to probability in Section 10.2. As they analyze probabilities, they will develop an understanding of conditional probability and more formal mathematical definitions of independence. With that students can determine if two categorical variables are associated with each other. To calculate and display probabilities, they will add the additional tool of two-way tables to their existing tools of area models and tree diagrams.	
Essential Questions and/or Enduring Understandings: 10.1 Relationships in a Circle 10.2 Conditional Probability 10.3 Permutations & Combinations	
ESSENTIAL Standards	**Learning Targets**

10.1	Students will learn that the perpendicular bisector of a chord passes through the center of the circle and will learn new circle-related vocabulary, such as major and minor arcs.
	Students will learn about the relationships between inscribed angles and the arcs that they intercept. Students will also learn the difference between arc measure and arc length.
	Students will learn that an angle inscribed in a semicircle measures 90°.
	Students will prove that opposite angles in an inscribed quadrilateral are supplementary.
	Students will develop different methods to find the length of a chord and will use the idea of similar triangles to find the relationships between the lengths created by two intersecting chords.
	Students will learn that a line tangent to a circle is perpendicular to the radius of the circle drawn to the point of tangency.
	Students will apply their knowledge of tangents, chords, angles, and arcs to solve problems involving circles.
	Students will consolidate their understanding of the relationships that exist between angles, arcs, chords, and tangents of a circle as they solve application problems.
10.2	Students will compare independent events with mutually exclusive events and will connect independence to the association of two variables.
	Students will calculate conditional probabilities from data arranged in relative frequency (probability) two-way tables (as opposed to frequency (count) two-way tables).
	Students will compare area models to two-way tables as methods for displaying probabilities.
	Students will learn the Multiplication Rule and an alternate definition for independence in probability situations.
	Students will apply their knowledge of independence to various situations.
10.3	Students will use the Fundamental Principle of Counting to count permutations and other outcomes when there are too many to list.
	Students will identify permutations and will develop two formulas for calculating the number of permutations.
	Students will describe the difference between permutations and combinations, learn how counting permutations can be

	a first step in counting combinations, and develop a method for counting combinations.
	Students will determine counting methods for situations that involve order and repetition, order and no repetition, no order with repetition, and no order without repetition.
	Students will solve challenging, multifaceted probability problems using the tools and techniques developed in this chapter.
NICE TO KNOW Standards	**Learning Targets**

UNIT 11: Solids & Circles	
Duration of Unit: 7 Day(s)	
Description of Unit: Students will learn how to identify the cross-sections of a solid and will investigate a special group of solids known as the Platonic Solids. As the word *geometry* literally means the "measurement of the Earth," it is only fitting that Section 11.2 focuses on developing the geometric tools that are used to learn more about the Earth. For example, by studying the height at which satellites orbit the Earth, students will get a chance to develop tools to work with the angle and arc measures that occur when two lines that are tangent to the same circle intersect each other.	
Essential Questions and/or Enduring Understandings: 11.1 Volume & Surface Area 11.2 Spheres	
ESSENTIAL Standards	**Learning Targets**
11.1	Students will build the five Platonic Solids (tetrahedron, octahedron, icosahedron, cube, and dodecahedron) and will understand why these are the only solids with faces that are congruent, regular polygons.
	Students will learn how to describe polyhedra using the number of faces (such as tetrahedron for any polyhedron with four faces) and will learn about dual polyhedra.

	Students will learn how a pyramid is defined and how to name it according to the shape of its base.
	Students will learn about slant height and finding the total surface area of a pyramid.
	Students will discover that the volume of a pyramid is one-third of the volume of a prism with the same base and height.
	Students will practice calculating the volume of a pyramid and will learn how to find the volume and surface area of a cone.
	Students will learn how to find the surface area and volume of a sphere.
11.2	Students will learn what a great circle is and will learn about a spherical coordinate system.
	Students will study the relationships between the measures of the arcs and angles formed when two lines that are tangent to the same circle intersect.
	Students will complete their study of circles in this course by finding the relationships between the measures of the angles and arcs intercepted by two intersecting secants or a secant and a tangent that intersect.
NICE TO KNOW Standards	**Learning Targets**

UNIT 12: Conics and Closure
Duration of Unit: 5 Day(s)
Description of Unit: As this course draws to a close, it is appropriate to reflect on what students have learned so far as they continue to see connections between topics in both algebra and geometry. Students will extend their geometric understanding of circles to write algebraic equations for circles. Then they will look at the cross-sections of a cone, called conic sections and learn about the geometric properties of parabolas. Students will find new connections between familiar geometric ideas and learn more special properties of familiar shapes.
Essential Questions and/or Enduring Understandings:

12.1 Conic Sections 12.2 Problem Solving	
ESSENTIAL Standards	**Learning Targets**
12.1	Students will learn how to find the equation of a circle graphed on coordinate axes.
	Students will complete the square to rewrite the equation of a circle from general form to graphing form.
	Students will identify the following cross-sections of a cone: point, line, absolute value, parabola, circle, ellipse, and hyperbola.
	Students will learn the geometric definition of a parabola (each point on a parabola is equidistant to the parabola's focus and directrix), and will investigate how the position of the focus and directrix affects the shape and direction of the parabola.
	Students will use the geometric definition of a parabola to graph various parabolas on focus-directrix paper.
	Students will derive the equation of a parabola using the geometric definition.
	Students will also be introduced to the geometric definition of an ellipse.
12.2	Students will learn that the quadrilateral formed by joining consecutive midpoints of any quadrilateral is a parallelogram.
	Students will review their understanding of polyhedra as they conjecture about the relationship of the number of faces, vertices, and edges of a basic polyhedron.
	Students will be introduced to phi (ϕ), the golden ratio, and will study several different contexts where phi arises.
	Students will review similarity, writing and solving quadratic equations, the angles of regular polygons, and the definitions of regular polyhedra.
	Students will find the areas of complex regions and will use probability to solve a challenging problem.
NICE TO KNOW Standards	**Learning Targets**
