Little Chute Math Department Grades 10-12, 1 Credit

In this course, students will learn to use new models and methods to think about problems as well as solve them. They will be developing powerful mathematical tools and learning new ways of thinking about and investigating situations. Students will be making connections, discovering relationships, figuring out what strategies can be used to solve problems, and explain your thinking. Learning to think in these ways and communicate about their thinking is useful in mathematical contexts, other subjects in school, and situations outside the classroom.

Timeframe	Unit	Instructional Topics
20 Day(s)	1-Investigations and Functions	1.1 Working Efficiently in Teams 1.2 Finding Multiple Representations
22 Day(s)	2-Transformations of Parent Graphs	2.1 Transforming Quadratic Functions 2.2 Transforming Other Parent Functions
18 Day(s)	3- Equivalent Forms	<ul><li>3.1 Rewriting Expressions and Equations</li><li>3.2 Rewriting and Simplifying Rational Expressions</li></ul>
18 Day(s)	4- Solving and Intersections	<ul><li>4.1 Writing and Solving Equations and Systems</li><li>4.2 Solving Inequalities and Systems of Inequalities</li></ul>
14 Day(s)	5- Inverses and Logarithms	5.1 Writing Inverses and Composite Functions 5.2 Investigating Logarithms
14 Day(s)	6- <u>3D Graphing and Logarithms</u>	<ul><li>6.1 Graphing and Solving 3-Variable Equations and Systems of Equations</li><li>6.2 Properties of Logarithms</li></ul>
16 Day(s)	7- Trigonometric Functions	7.1 Trig Functions and The Unit Circle 7.2 Transforming Trig Functions
14 Day(s)	8- <u>Polynomials</u>	<ul><li>8.1 Graphing and Writing Equations of Polynomial Functions</li><li>8.2 Complex Numbers</li><li>8.3 Dividing Polynomials and Solving Equations</li></ul>
12 Day(s)	9- Randomization and Normal Distributions	9.1 Collecting Data in a Survey

#### Scope and Sequence

		9.2 Randomness in Surveys and Experiments 9.3 Relative Frequency Histograms
15 Day(s)	10- <u>Series</u>	10.1 Arithmetic Sequences and Series 10.2 Geometric Sequences and Series 10.3 Pascal'sTriangle and the Binomial Theorem
11 Day(s)	11- Simulating Sampling Variability	<ul><li>11.1 Simulations of Probability</li><li>11.2 Conducting a Statistical Hypothesis Test</li><li>11.3 Analyzing Decisions and Strategies</li></ul>
14 Day(s)	12- Analytic Trigonometry	12.1 Solving Trigonometric Equations 12.2 Trigonometric Identities

# **UNIT 1: Investigations and Functions**

# Duration of Unit: 20 Day(s)

**Description of Unit:** This chapter will introduce you to the ways you will be working as well as several of the big ideas in this course. You will share your current mathematical knowledge with your study team as you work together to solve problems. Some of these ideas you will revisit later in the course and connect to new mathematical ideas. You will learn to work with a graphing calculator to help you discover qualities of functions and systems of functions.

#### **Essential Questions and/or Enduring Understandings:**

1.1 Working Efficiently in Teams

1.2 Finding Multiple Representations

ESSENTIAL Standards	Topics	Learning Targets
	1.1	Students will be challenged to work together as a team to share mathematical ideas and to justify their strategies as they represent geometric objects and order a series of connected functions to create a desired output.
		Students will draw complete graphs of functions and identify possible inputs, outputs, and key points for describing those graphs. Students will use a graphing calculator and will develop presentation

		skills.
		Students will identify the domain and range of functions while improving their graphing-calculator skills.
		Students will find points of intersection using multiple representations and will learn how to use the [TABLE], [TBLSET], and [CALC] functions of their graphing calculators.
	1.2	Students will investigate a function defined by a geometric relationship and will generate multiple algebraic representations for the function.
		Students will develop their understanding of what it means to investigate a function as they investigate a family of hyperbolas.
		Students will identify commonalities in a family of linear functions and will determine whether relationships in tables and situations are linear.
		Students will investigate a nonlinear function.
NICE TO KNOW Standards		Learning Targets

# **UNIT 2: Transformations of Parent Graphs**

### Duration of Unit: 22 Day(s)

**Description of Unit:** In the first section of Chapter 2, you will learn how to change the equation of a parabola to make it fit a set of nonlinear data. After you learn how to stretch, compress, reflect, and shift the graph of  $f(x) = x^2$ , you will be able to create a variety of parabolic shapes and sizes. You will learn that a graph's transformations are clearly recognizable when its equation is written in graphing form. Understanding this form will help you learn how to rewrite equations so that they are easier to graph. You will also use the quadratic family of functions to model physical situations, such as the arc of a jumping rabbit and the path of a soccer ball. In Section 2.2, you will apply these same types of transformations to other parent functions.

# **Essential Questions and/or Enduring Understandings:** 2.1 Transforming Quadratic Functions 2.2 Transforming Other Parent Functions

ESSENTIAL Standards	Topics	Learning Targets
	2.1	Students will collect non-linear data, fit an equation to their data, and use their equation to make predictions.
		Students will connect transformations of parabolas with their equations in graphing form.
		Students will graph quadratic equations without making tables. Students will rewrite quadratic equations from standard form into graphing form.
		Students will review, use, and compare two methods for rewriting quadratic equations from standard form to graphing form, averaging the intercepts and completing the square.
		Students will learn how to write quadratic equations for situations using the graphing form of the parabola $y = a(x - h)^2 + k$ . Specifically, students will develop an algebraic strategy for finding the value of the stretch factor, a.
	2.2	Students will transform the graphs of y = b <sup>x</sup> , y = $\frac{1}{x}$ , y = $\sqrt{x}$ , y = $ x $ , and y = x <sup>3</sup>
		Students will identify the point (h, k) for graphs of parabolic, hyperbolic, cubic, absolute value, exponential, and square root functions and relate it to the point-slope form of a line.
		Students investigate one more transformation, $f(-x)$ , the reflection of $f(x)$ across the y-axis. Then they will compare $f(-x)$ and $-f(x)$ , for a variety of parent graphs and develop the definitions for even and odd functions.
		Students will use what they know about transforming parabolas to make conjectures about transforming non-functions, specifically a circle.
		Students will use what they know about transformations to relocate and reorient a piecewise-defined function.
NICE TO KNOW		Learning Targets

Standards	

# **UNIT 3: Equivalent Forms**

**Duration of Unit: 18 Day(s)** 

**Description of Unit:** In this chapter, you will focus on rewriting expressions in order to have more useful equivalent forms. You will remind yourself what it means for two expressions or equations to be equivalent. You will then rewrite equations to solve them more easily. Another focus of this chapter is learning how to combine algebraic fractions (called "rational expressions") and expressions with exponents. By using the special properties of the number 1 and the meaning of exponents, you will be able to rewrite long, complicated expressions into simpler forms. You will then multiply, divide, add, and subtract rational expressions.

#### **Essential Questions and/or Enduring Understandings:**

3.1 Rewriting Expressions and Equations

3.2 Rewriting and Simplifying Rational Expressions

ESSENTIAL Standards	Topics	Learning Targets
	3.1	Students will identify equivalent expressions and develop algebraic strategies for demonstrating equivalence.
		Students will use an area model to multiply expressions. They will factor expressions and demonstrate equivalence.
		Students will rewrite and solve equations and systems of equations, sometimes using substitution.
	3.2	Students will explore the graphs of several rational functions in order to visualize some of the effects of dividing by polynomials.

	Students will analyze and compare rational expressions.
	Students will understand how to multiply and divide rational expressions and will continue to learn how to simplify rational expressions.
	Students will learn how to add and subtract rational expressions.
	Students will consider problems involving all four operations with rational expressions and consolidate their understanding. They will learn how to check their work when simplifying rational expressions.
NICE TO KNOW Standards	Learning Targets

# **UNIT 4: Solving and Intersections**

Duration of Unit: 18 Day(s)

**Description of Unit:** This chapter begins with a focus on two ways to solve equations and systems of equations: algebraically and graphically. You will build on your understanding of solving and solutions from previous courses to gain a broader and stronger understanding of the meaning of solutions. In Section 4.2, you will expand your understanding of solving and solutions to include inequalities. You will solve problems designed to illustrate how inequalities might be used for more complicated applications.

### **Essential Questions and/or Enduring Understandings:**

4.1 Writing and Solving Equations and Systems

4.2 Solving Inequalities and Systems of Inequalities

ESSENTIAL Standards	Topics	Learning Targets
	4.1	Students will solve a variety of equations and discuss different methods for solving them. Students will justify their strategies and develop a strategy for checking their solutions.

		Students will use graphs to validate algebraic solutions and to approximate solutions when no algebraic method is available, and they will use two different methods to solve one-variable equations graphically.
		Students will solve systems of linear and non-linear equations using multiple strategies. Students will determine the number of solutions for systems and interpret solutions graphically.
		Students will use their problem-solving skills to write equations and find solutions for real-life applications.
	4.2	Students will extend what they learned about solving systems of equations graphically to solving systems of inequalities.
		Students will apply systems of linear inequalities to solve a problem involving an everyday situation.
		Students will apply systems of linear inequalities to solve a problem involving an everyday situation.
		Students will consider two functions and identify the relationships between the functions and the system from which they come.
NICE TO KNOW Standards		Learning Targets

# **UNIT 5: Inverse and Logarithms**

#### **Duration of Unit: 14 Day(s)**

**Description of Unit:** In this chapter you investigate some new functions that "undo" each other. You will learn about inverse relationships and investigate the relationships between functions and their inverses. You will also learn about compositions of functions. In Section 5.2, you will find the inverses of many parent graphs and add them to the tools you have for working with parent graphs. You will find inverses for exponential functions, which are called logarithmic functions. You will then investigate this family of functions and transform its

# graphs.

# **Essential Questions and/or Enduring Understandings:** 5.1 Writing Inverses and Composite Functions

- 5.2 Investigating Logarithms

ESSENTIAL Standards	Topics	Learning Targets
	5.1	Students will learn to find equations that "undo" functions, and will develop strategies to justify that each equation undoes the other. They will graph functions along with their inverses and make observations about the relationships between the graphs.
		Students will learn to find equations that "undo" functions, and will develop strategies to justify that each equation undoes the other. They will graph functions along with their inverses and make observations about the relationships between the graphs.
		Students will use their ideas about switching x- and y-values to learn how to find an inverse algebraically. Students will learn about compositions of functions and will use compositions $f(g(x))$ and $g(f(x))$ to test algebraically whether two functions are inverses of each other.
	5.2	Students will apply their strategies for finding inverses to parent graph equations. They will begin to think of the inverse function for $y = 3^x$ as "the exponent for 3 that will give x," or "What I would get if I could change x = $3^y$ into y = form?" as a precursor for learning about logarithms.
		Students will define the term logarithm as the inverse exponential function.
		Students will develop methods to graph logarithmic functions with different bases. They will rewrite logarithmic equations as exponential equations and find inverses of logarithmic functions.
		Students look into the base of the log key on their calculator. Students will extend their knowledge of general equations for parent functions to transform the graph of $y = \log x$ .
NICE TO KNOW Standards		Learning Targets

# UNIT 6: 3D Graphing and Logarithms

### Duration of Unit: 14 Day(s)

**Description of Unit:** In this chapter, you will learn to extend your mathematical thinking to three dimensions and you will further your understanding of logarithms, which will give you the tools to solve a murder mystery. In the first section, you will expand your understanding of graphing equations and systems of equations to three dimensions and you will broaden your understanding of solutions to include solutions to systems in three dimensions. In Section 6.2, you will return to logarithms to learn more about their properties and why they are useful. You will construct an exponential function to model a situation, and you will use logarithms to solve a mathematical murder mystery.

- 6.1 Graphing and Solving 3-Variable Equations and Systems of Equations
- 6.2 Properties of Logarithms

ESSENTIAL Standards	Topics	Learning Targets
	6.1	Students will create and use a model to locate points in three-dimensional space and will plot points in three dimensions on isometric dot paper.
		Students will graph planes on three-dimensional axes.
		Students will investigate the graphs of systems of equations with three variables. They will find points that lie on two planes simultaneously.
		Students will develop an algebraic strategy to solve systems of three equations with three variables. Students will also determine the different ways three planes can intersect and will investigate the graphs of three-dimensional systems.
		Students will work in teams to find the equation of a quadratic function $y = ax^2 + bx + c$ that passes through three given points when graphed.
	6.2	Students develop the Power Property of Logs and use it to develop an efficient method to solve exponential equations in $a^x = b$ form.
		Students will learn the Product and Quotient Properties of logs and how to rewrite equations with

	different bases.
	Students will develop strategies for finding the equation of an exponential function given two points and an asymptote.
	Students will apply their knowledge of exponential functions to solve a murder mystery.
NICE TO KNOW Standards	Learning Targets

# **UNIT 7: Trigonometric Functions**

# **Duration of Unit: 16 Day(s)**

**Description of Unit:** This chapter begins with an experiment that will generate a new curve. You will then explore the relationship between right-triangle trigonometry and this new curve. You will be introduced to a new representation that is useful for the study of trigonometric functions: a unit circle. You will also learn how to use radians instead of degrees to describe angles. In the second section of this chapter, you will transform trigonometric functions and find general equations for them. You will also learn about a new property that is characteristic of trigonometric functions called a period. Then you will write equations for the curve that you generated in the experiment at the beginning of the chapter.

- 7.1 Trig Functions and The Unit Circle
- 7.2 Transforming Trig Functions

ESSENTIAL Standards	Topics	Learning Targets
	7.1	Students will conduct an experiment with a pendulum that will result in a sine curve. They will predict ways to change the shape of the curve.
		Students will use experimental data generated from measuring the heights of right triangles to create a

		sine graph.
		Students will develop an understanding of reference angles and will explore the connections between the sine graph and the unit circle.
		Students will use the cosine function to calculate horizontal distances in a unit circle. They will draw conclusions about the relationships between the sine and cosine of an angle and the sine and cosine graphs.
		Students will use the definition of a radian to construct an angle that measures one radian. They will also determine the number of radians in a full circle.
		Students will use their knowledge of special triangles and reference angles to label the exact values of the coordinates on a unit circle.
		Students will recognize the connection between the tangent of an angle in the unit circle and the slope of the corresponding radius. They will use a unit circle to generate a graph of $t(\theta) = tan\theta$ .
	7.2	Students will apply their understanding of transforming parent graphs to the sine and cosine functions. They will generate general equations for the family of sine and cosine functions of the form $y = asin(x - h) + k$ .
		Students will determine the placement of the parameter b in the general equation for sine and cosine. They will also identify the period of cyclic situations.
		Students will find equations for transformed sine curves and will graph transformed sine functions.
		Students will consolidate their understanding of the connections between cyclic graphs and their equations. They will also practice graphing equations and writing equations from graphs. Finally, they will determine that sine and cosine functions are just horizontally shifted versions of each other.
NICE TO KNOW Standards		Learning Targets

# **UNIT 8: Polynomials**

# Duration of Unit: 14 Day(s)

**Description of Unit:** In this chapter you will expand your knowledge of families of functions to include polynomial functions. As you investigate the equation  $\leftrightarrow$  graph connection for polynomials, you will learn how to search for factors (which can help you find *x*-intercepts) and how to use division to find additional factors. When you investigate the graphs of polynomials and systems involving polynomials, you will see many that appear not to intersect. As you investigate these systems further, you will learn about imaginary and complex numbers. In the last section of the chapter, you will apply your knowledge of polynomials to model some of the attractions at a county fair.

- 8.1 Graphing and Writing Equations of Polynomial Functions
- 8.2 Complex Numbers
- 8.3 Dividing Polynomials and Solving Equations

ESSENTIAL Standards	Topics	Learning Targets
	8.1	Students will describe the graph of a polynomial given its equation in factored form.
		Students will consolidate, generalize, and explain their findings from the polynomial investigation that they did in Lesson 8.1.1.
		Students will write exact equations for the graphs of polynomial functions given the x-intercepts and one additional point.
	8.2	Students will solve equations using imaginary and complex numbers.
		Students will be introduced to complex conjugates by solving quadratic equations and then will learn how to write the equations for quadratic functions given the roots. Students will practice operations with complex numbers.
		Students will be introduced to the complex plane as a way to visualize complex numbers and complex roots for quadratic functions. They will calculate the absolute value of a complex number. In addition, students will start to investigate the number of linear and quadratic factors a polynomial can have.

	8.3	Students will use polynomial division to find factors of polynomials.
		Students will use the Integral Zero Theorem, polynomial division and the Factor Theorem to find integral roots and to find all roots of polynomials with degree greater than two.
		Students will use their knowledge of polynomial functions and graphs to maximize volume of a tank.
NICE TO KNOW Standards		Learning Targets

# **UNIT 9: Randomization and Normal Distributions**

Duration of Unit: 12 Day(s)

**Description of Unit:** In this chapter you will learn some basic techniques of performing opinion surveys along with their limitations and pitfalls. You will learn why randomness is a cornerstone of statistical studies. In the last section of the chapter, you will create a histogram with percentages called a relative frequency histogram. You will learn a new way to describe the shape of a distribution, and use it to model certain distributions.

- 9.1 Collecting Data in a Survey
- 9.2 Randomness in Surveys and Experiments
- 9.3 Relative Frequency Histograms

ESSENTIAL Standards	Topics	Learning Targets
	9.1	Students will write research questions and consider issues of bias as they write survey items to investigate those questions.
		Students will compare the representative nature of samples selected using intentional choice with

		those selected randomly. They will learn that random sampling produces samples that, in the aggregate, represent populations better than samples intentionally selected by humans.
		Students will consider populations represented by particular convenience samples. They will incorporate some level of random selection into their own plans for sampling.
	9.2	Students will discover the importance of randomization in an experiment.
		Students will practice the main concepts of this chapter as they think critically about and analyze the conclusions for various fictitious studies.
	9.3	Students will create a relative frequency histogram that displays percentages instead of counts.
		Students will create a relative frequency histogram that displays percentages instead of counts. They will then model bell-shaped data with a normal distribution and use the model to calculate proportions.
		Students will continue their exploration of using a normal distribution to model single-peaked symmetrical data. Based on their normal distributions, students will predict into which percentiles various subjects fall.
NICE TO KNOW Standards		Learning Targets

# **UNIT 10: Series**

#### **Duration of Unit: 15 Day(s)**

**Description of Unit:** In this chapter you will revisit and add to what you already know about arithmetic and geometric sequences. In Sections 10.1 and 10.2 you will use what you know about sequences and multiple representations to write series and find their sums. In Section 10.3 you will use what you learned about combinations to develop the Binomial Theorem, which is useful for simplifying some algebraic manipulations, as well as solving some probability problems.

# **Essential Questions and/or Enduring Understandings:**

10.1 Arithmetic Sequences and Series

10.2 Geometric Sequences and Series

10.3 Pascal's Triangle and the Binomial Theorem

ESSENTIAL Standards	Topics	Learning Targets
	10.1	Students will be introduced to arithmetic series and will learn how to distinguish series from sequences. Students will begin to develop strategies to find sums of arithmetic series.
		Students will generalize a graphical method for finding the sum of an arithmetic series and then apply it.
		Students will find the sums of series that have unspecified numbers of terms and will learn how known series can be combined to form new series.
		Students will see an algebraic method for finding the sum of an arithmetic series. They will be introduced to summation notation for arithmetic series.
	10.2	Students will calculate sums of geometric series.
		Students will find sums of infinite geometric series.
	10.3	Students will make a table of combinations which, when reoriented is Pascal's Triangle. They will connect the numbers in the rows of the triangle with the expansion of binomial expressions.
		Students will explore the origins of the transcendental number, e.
NICE TO KNOW Standards		Learning Targets

# **UNIT 11: Simulating Sampling Variability**

**Duration of Unit: 11 Day(s)** 

**Description of Unit:** In this chapter, you will use computer simulations to model complex probabilities—probabilities that are often too difficult to compute mathematically. Then you will simulate sample-to-sample variability. Your knowledge of sample-to-sample variability will help you place a margin of error on your predictions about certain characteristics of populations and will help you make statistical decisions.

- 11.1 Simulations of Probability
- 11.2 Conducting a Statistical Hypothesis Test
- 11.3 Analyzing Decisions and Strategies

ESSENTIAL Standards	Topics	Learning Targets
	11.1	Students will toss coins to simulate the probability of giving birth to boys and girls. They will then use their calculators to make repetitions of the simulation less tedious and check results with the theoretical probability.
		Students will use technology to simulate the number of streaks they can expect in a random process. They will do a computer simulation of wait time—how many trials they can expect to conduct until a particular event occurs.
		Students will determine the natural sample-to-sample variability when taking a random sample from the population. Students will determine the margin of error on a sample proportion.
	11.2	Students will continue their investigations into sample-to-sample variability by conducting a hypothesis test—determining whether a claim about the population is supported by their survey using two different sample sizes.
		Students will simulate sample-to-sample variability to determine if it is plausible that two treatments in an experiment are truly different.
		Students will use simulations to decide whether or not a manufactured part is within typical quality

		specifications.
		Students will simulate a quality control process.
	11.3	Students will apply probabilities to analyze decisions and strategies. The two problems in this lesson are applications of conditional probability in situations where the results are counterintuitive. It is often difficult to convince people of these results' accuracy.
NICE TO KNOW Standards		Learning Targets

# **UNIT 12: Analytic Trigonometry**

### Duration of Unit: 14 Day(s)

**Description of Unit:** In this chapter, you will continue your study of trigonometry, this time investigating solutions to trigonometric equations. You will learn about three new trigonometric ratios (secant, cosecant, and cotangent) and their corresponding functions. By the end of this chapter, you will be able to solve a wide variety of trigonometric equations. You will make statements, based on the unit circle and the graph, about how many solutions there are and why.

- 12.1 Solving Trigonometric Equations
- 12.2 Trigonometric Identities

ESSENTIAL Standards	Topics	Learning Targets
	12.1	Students will determine if trigonometric equations are always true, sometimes true, or never true. They will broaden their understanding of the meaning of solutions.
		Students will solve trigonometric equations and represent the solutions graphically, algebraically, and on a unit circle. Students will also determine the number of solutions to given equations.

		Students will graph the inverses of trigonometric functions and will recognize the restricted domains that allow the inverses to also be functions.
		Students will graph the reciprocal trigonometric functions and solve equations involving these functions.
	12.2	Students will discover trigonometric identities graphically and use those identities to rewrite and solve equations.
		Students will write algebraic proofs for trigonometric identities.
		Students will develop the Angle Sum and Difference Identities using relationships on the unit circle.
NICE TO KNOW Standards		Learning Targets