AP Calculus

Little Chute
Mathematics
Grades 9 - 12, 1.5 Credits

Course Overview

Calculus is a course that investigates the key mathematical concepts of the function, the limit, the derivative, and the integral. Each of these concepts will be discussed from graphical, numerical, and algebraic perspectives. These concepts will be used to explore applications within the fields of mathematics, the physical sciences, engineering, the social sciences, and the biological sciences.

Scope and Sequence

Timeframe	Unit	Instructional Topics
19 Day(s)	1- Functions: A Pre-Calculus Perspective Functions	1.1- The Four Methods of Expressing Functions 1.2- Linear Functions 1.3 Exponential and Logarithmic Functions 1.4- Power Functions 1.5- Trigonometric Functions 1.6- Polynomial and Rational Functions
9 Day(s)	2- Limits: Crossing the Bridge	2.1- Estimating Limits Graphically and Numerically 2.2- Applying Properties of Limits 2.3- Calculating Limits Algebraically 2.4- Continuity 2.5- One-Sided Limits and Limits Involving Infinity 2.6- The Intermediate Value Theorem and The Extreme Value Theorem
12 Day(s)	3- The Derivative	3.1- The Derivative at a Point 3.2- Interpreting Derivatives 3.3- Derivative Functions 3.4- Relationships Between a Function and its Derivatives 3.5- Position, Velocity, and Acceleration
25 Day(s)	4- Functions: A Calculus Perspective	4.1- Differentiation Rules of Basic Functions 4.2- Differentiation Rules for Combinations of Basic Functions 4.3- Implicit Differentiation 4.4- Analyzing Functions Using Calculus 4.5- Curve Sketching
17 Day(s)	5- Applications of Derivatives	5.1- Related Rates

		5.2- Optimization 5.3- Rolle's Theorem and the Mean Value Theorem
25 Day(s)	6- The Integral	6.1- Riemann Sums and The Definite Integral 6.2- Areas 6.3- The Fundamental Theorem of Calculus 6.4- Integral of Rate of Change Equals Total Change 6.5- Anti-Derivatives of Basic Functions 6.6- Volumes 6.7- Arc Lengths 6.8- Separable Differential Equations
25 Day(s)	7- AP Review	7.1- Analyze a variety of AP free response programs and classify problems into discrete categories 7.2- Analyze examples of AP multiple choice problems

UNIT 1: Functions: A Pre-Calculus Perspective Functions

Duration of Unit: 19 Day(s)

Description of Unit: In this unit, we will be reviewing a variety of important pre-calculus concepts that are necessary prerequisites for understanding Calculus. We will discuss the definition of a function and the four ways of representing a function: verbally, graphically, numerically, and algebraically. We will then discuss concepts that can be applied to functions in general including function notation, evaluating functions, composition of functions, inverse functions, and symmetry of functions. After we have reviewed these function basics, we will then analyze specific families of functions including linear functions, exponential functions, power functions, logarithmic functions, trigonometric functions, rational functions, and polynomial functions.

Essential Questions and/or Enduring Understandings:
1.1- The Four Methods of Expressing Functions
1.2- Linear Functions
1.3 Exponential and Logarithmic Functions
1.4- Power Functions
1.5- Trigonometric Functions
1.6- Polynomial and Rational Functions

ESSENTIAL Standards	Topics	Learning Targets
	1.1	Students will apply the concept of the inverse of a function.
		Students will evaluate a function for a given value of the domain.
		Students will identify the domain, range,independent variable, and dependent variable for a given function.
		Students will interpret the meaning of functions with respect to units.
		Students will solve problems involving the composition and decomposition of functions.
		Students will translate a function from a given form to any of the other three forms.
	1.2	Students will calculate the slope and y-intercept of a line.

		Students will define slope and y-intercept.
		Students will write the equation of a linear function given a graph, numerical representation, or written description of the function.
	1.3	Students will apply properties of exponents and logarithms.
		Students will determine the doubling time or half life of an exponential function.
		Students will determine the growth rate multiplier for an exponential function represented by a table, graph, or written description.
		Students will determine the value of Po given an exponential function represented by a table, graph, or written description.
		Students will graph exponential and logarithmic functions.
		Students will solve exponential equations using logarithms.
		Students will write the equation of an exponential function given a graph, numerical representation, or written description of the function.
	1.4	Students will discuss the concept of dominance as it relates to exponential and power functions.
		Students will graph power functions with positive integral exponents, negative integral exponents, and fractional exponents.
	1.5	Students will determine the amplitude, period, vertical shift, and phase shift of a trigonometric function.
		Students will solve trigonometric equations using inverse trigonometric functions.
		Students will write a formula for a trigonometric function given a graph or verbal description of a trigonometric function.
	1.6	Students will determine the general shape of a polynomial function by analyzing the degree of the function and the leading coefficient.
		Students will identify the equations of the horizontal and vertical asymptotes of a rational function.
		Students will solve polynomial and rational equations.

		Students will write the equation of a polynomial function given a graph of the function.
NICE TO KNOW Standards		**Learning Targets**

UNIT 2: Limits: Crossing the Bridge

Duration of Unit: 9 Day(s)

Description of Unit: The concept of the limit is what allows us to make a connection between pre-calculus mathematics and the mathematical techniques of calculus. The two main topics of calculus, derivatives and integrals, are defined in terms of limits. In this unit we will learn what limits are and how they actually work. We will emphasize an intuitive understanding of limits, but we will also engage in a somewhat formal discussion of limits. We will then learn about the properties that limits possess. We will use these properties to discuss a variety of techniques for evaluating limits algebraically. We will also see how limits are required to define an important mathematical concept known as continuity. Finally, we will see what happens when we work with limits involving infinity.

Essential Questions and/or Enduring Understandings:
2.1- Estimating Limits Graphically and Numerically
2.2- Applying Properties of Limits
2.3- Calculating Limits Algebraically
2.4- Continuity
2.5- One-Sided Limits and Limits Involving Infinity
2.6- The Intermediate Value Theorem and The Extreme Value Theorem

ESSENTIAL Standards	Topics	Learning Targets
	2.1	Students will estimate the value of a limit of a function by looking at a graph of the function.
		Students will estimate the value of a limit of a function given the function in table form.
	2.2	Students will recognize situations in which the various properties apply.

		Students will state the important properties of limits.
	2.3	Students will apply cancellation techniques for evaluating limits algebraically.
		Students will apply rationalization techniques for evaluating limits algebraically.
	2.4	Students will discuss the concept of continuity informally.
		Students will relate the formal definition of continuity to informal conceptions of continuity.
	2.5	Students will define one-sided limits.
		Students will discuss limits that approach infinity as x approaches a given number.
		Students will discuss the meaning of limits as x approaches infinity.
	2.6	Students will discuss the importance of continuity to the Intermediate Value Theorem and Extreme Value Theorem.
		Students will state the Intermediate Value Theorem and Extreme Value Theorem.
NICE TO KNOW Standards		

UNIT 3: The Derivative

Duration of Unit: 12 Day(s)

Description of Unit: After our pre-calculus review and our discussion of limits, we are now ready to study the first major concept of Calculus: The Derivative. We will define the concept of the derivative within a specific practical context, and then we will see how the concept can be generalized to help analyze functions in a wide variety of contexts. We will learn algebraic, graphical, and numerical techniques for evaluating derivatives, and we will analyze the relationship between a function and its derivative functions.

Essential Questions and/or Enduring Understandings:
3.1- The Derivative at a Point
3.2- Interpreting Derivatives
3.3- Derivative Functions
3.4- Relationships Between a Function and its Derivatives
3.5- Position, Velocity, and Acceleration

ESSENTIAL Standards	Topics	Learning Targets
	3.1	Students will define the concept of a derivative in two different ways by solving the tangent line problem.
		Students will evaluate derivatives of functions given numerically, algebraically, and graphically using numerical techniques including the use of the graphing calculator.
		Students will discuss the concept of velocity as the derivative of the position function.
	3.2	Students will discuss the interpretation of a derivative as a rate of change.
		Students will identify units of a derivative for a given function.
	3.3	Students will determine the derivative function for a function given as a table.
		Students will determine the derivative function for a given algebraic function by using the definition of a derivative.
		Students will sketch the graph of the derivative function for a function given graphically.
	3.4	Students will discuss the relationship between the concavity of a function and the sign of the second derivative function.
		Students will discuss the relationship between the increasing and decreasing behavior of a function and the sign of the derivative function.
	3.5	Students will discuss the concept of acceleration as the derivative of the velocity function.
		Students will discuss the concept of velocity as the derivative of the position function.
NICE TO KNOW Standards		**Learning Targets**

UNIT 4: Functions: A Calculus Perspective

Duration of Unit: 25 Day(s)

Description of Unit: Now that we have reviewed functions from a pre-calculus perspective, discussed the bridging concept of a limit, and been introduced to the concept of the derivative, we are now ready to analyze functions using techniques of calculus. In this unit we will be learning and practicing a variety of "shortcuts" for finding derivative functions. At the same time we will learn how to use these derivative functions to help obtain a more in-depth understanding of functions. In the end, we will see that we can develop a very clear picture of what the graph of a function looks like when we combine the techniques that we have learned.

Essential Questions and/or Enduring Understandings:
4.1- Differentiation Rules of Basic Functions
4.2- Differentiation Rules for Combinations of Basic Functions
4.3- Implicit Differentiation
4.4- Analyzing Functions Using Calculus
4.5- Curve Sketching

ESSENTIAL Standards	Topics	Learning Targets
	4.1	Students will find the derivative of exponential functions.
		Students will find the derivative of inverse trigonometric functions.
		Students will find the derivative of logarithmic functions.
		Students will find the derivative of power functions.
		Students will find the derivative of trigonometric functions.
	4.2	Students will find the derivatives of products and quotients of functions.
		Students will find the derivative of sums and differences of functions.
		Students will identify the location of roots, critical points, and points of inflection.

	4.3	Students will describe the difference between implicit and explicit functions.
		Students will discuss the steps involved in the process of implicit differentiation.
	4.4	Students will identify critical points and critical values of a function.
		Students will identify intervals where a function is increasing, decreasing, concave up, and concave down.
		Students will identify the location of points of inflection of a function.
	4.5	Students will identify the location of roots, critical points, and points of inflection.
NICE TO KNOW Standards		**Learning Targets**

UNIT 5: Applications of Derivatives

Duration of Unit: 17 Day(s)

Description of Unit: In this unit we will study on applications of differentiation. We will focus on two main applications: related rates and optimization. We will use many of the concepts that we have studied from previous units to help develop the theory behind these applications.

Essential Questions and/or Enduring Understandings:
5.1- Related Rates
5.2- Optimization
5.3- Rolle's Theorem and the Mean Value Theorem

ESSENTIAL Standards	Topics	Learning Targets
	5.1	Students will describe the steps used to solve a related rates problem.

		Students will identify primary and secondary equations for a related rates problem.
	5.2	Students will identify a quantity to be optimized.
		Students will identify the appropriate procedure to locate the global maximum and/or minimum of the quantity to be optimized.
		Students will use secondary equations to write the primary equation in terms of a single variable.
		Students will write a primary equation for the quantity to be optimized.
	5.3	Students will determine whether Rolle's Theorem or The Mean Value Theorem applies to a given situation.
		Students will state the conclusion of Rolle's Theorem or The Mean Value Theorem given that the conditions of the theorem apply.
NICE TO KNOW Standards		**Learning Targets**

UNIT 6: The Integral

Duration of Unit: 25 Day(s)

Description of Unit: The integral is the second major concept of Calculus. Our study of the integral will be analogous to our study of the derivative. We will introduce the concept within a specific context and then generalize the concept to see how it can be applied and interpreted in a wide variety of contexts. We will learn how to find integrals using shortcuts similar to those that we use to find derivatives. In fact, the most important concept that we will study is the relationship between derivatives and integrals as described by The Fundamental Theorem of Calculus. We will also focus on a variety of applications of integration including finding volumes, lengths of arcs, and solving differential equations.

Essential Questions and/or Enduring Understandings:
6.1- Riemann Sums and The Definite Integral
6.2- Areas
6.3- The Fundamental Theorem of Calculus

6.4- Integral of Rate of Change Equals Total Change
6.5- Anti-Derivatives of Basic Functions
6.6- Volumes
6.7- Arc Lengths
6.8- Separable Differential Equations

ESSENTIAL Standards	Topics	Learning Targets
	6.1	Students will define a Riemann Sum.
		Students will discuss the notations used for Riemann sums and definite integrals and the relationships between them.
		Students will evaluate integrals of functions given numerically, graphically, and algebraically.
		Students will informally define a definite integral as an area "under" a curve.
	6.2	Students will calculate the area between a curve and the x-axis.
		Students will find the area between two curves.
	6.3	Students will discuss an informal proof of the Fundamental Theorem of Calculus.
		Students will discuss the implications of the Fundamental Theorem of Calculus.
		Students will evaluate definite integrals by finding antiderivatives.
	6.4	Students will apply appropriate techniques to evaluate an integral in a given situation.
		Students will determine the appropriate units for an integral in a given situation.
		Students will use integrals to determine the average value of a given function.
	6.5	Students will find a specific anti-derivative using initial conditions.
		Students will find anti-derivatives using the technique of substitution.
		Students will find general anti-derivatives that follow from derivatives of basic functions.

	6.6	Students will find the volume of a solid formed by rotating a given region around the x or y axis.
		Students will find the volume of a solid object with known cross sections.
	6.7	Students will Discuss the derivation of the formula for arc length.
	6.8	Students will Create a mathematical model given a verbal description of a differential equation.
		Students will Describe the steps that are used to solve a separable differential equation.
NICE TO KNOW Standards		**Learning Targets**